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Direct Stability Methods

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Direct methods of stability analysis determine the transient stability (as defined in [Chapter 7](#) and described in [Chapter 8](#)) of power systems without explicitly obtaining the solutions of the differential equations governing the dynamic behavior of the system. The basis for the method is Lyapunov's second method, also known as Lyapunov's direct method, to determine stability of systems governed by differential equations. The fundamental work of A.M. Lyapunov (1857–1918) on stability of motion was published in Russian in 1893, and was translated into French in 1907 (Lyapunov, 1907). This work received little attention and for a long time was forgotten. In the 1930s, Soviet mathematicians revived these investigations and showed that Lyapunov's method was applicable to several problems in physics and engineering. This revival of the subject matter has spawned several contributions that have led to the further development of the theory and application of the method to physical systems.

The following example motivates the direct methods and also provides a comparison with the conventional technique of simulating the differential equations governing the dynamics of the system. [Figure 11.1](#) shows an illustration of the basic idea behind the use of the direct methods. A vehicle, initially at the bottom of a hill, is given a sudden push up the hill. Depending on the magnitude of the push, the vehicle will either go over the hill and tumble, in which case it is unstable, or the vehicle will climb only part of the way up the hill and return to a rest position (assuming that the vehicle's motion will be damped), i.e., it will be stable. In order to determine the outcome of disturbing the vehicle's equilibrium for a given set of conditions (mass of the vehicle, magnitude of the push, height of the hill, etc.), two different methods can be used:

1. Knowing the initial conditions, obtain a time solution of the equations describing the dynamics of the vehicle and track the position of the vehicle to determine how far up the hill the vehicle will travel. This approach is analogous to the traditional time domain approach of determining stability in dynamic systems.
2. The approach based on Lyapunov's direct method would consist of characterizing the motion of the dynamic system using a suitable Lyapunov function. The Lyapunov function should satisfy certain sign definiteness properties. These properties will be addressed later in this subsection. A natural choice for the Lyapunov function is the system energy. One would then compute the

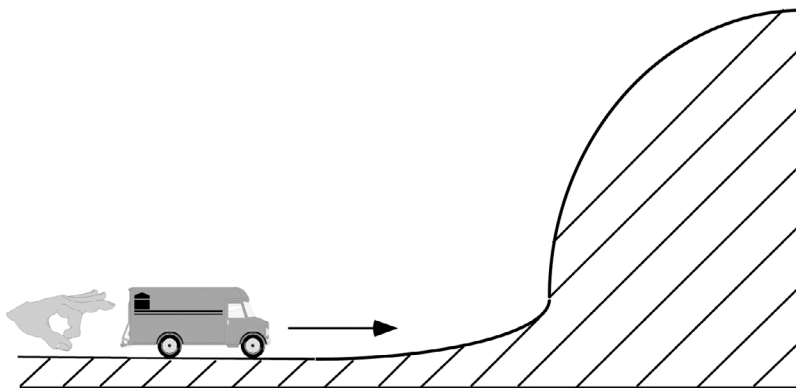


FIGURE 11.1 Illustration of idea behind direct methods.

energy injected into the vehicle as a result of the sudden push, and compare it with the energy needed to climb the hill. In this method, there is no need to track the position of the vehicle as it moves up the hill.

These methods are simple to use if the calculations involve only one vehicle and one hill. The complexity increases if there are several vehicles involved as it becomes necessary to determine (a) which vehicles will be pushed the hardest, (b) how much of the energy is imparted to each vehicle, (c) which direction will they move, and (d) how high a hill must they climb before they will go over the top.

The simple example presented here is analogous to analyzing the stability of a one-machine-infinite-bus power system. The approach presented here is identical to the well-known equal area criterion (Kimbark, 1948; Anderson and Fouad, 1994) which is a direct method for determining transient stability for the one-machine-infinite-bus power system. For a more detailed discussion of the equal area criterion and its relationship to Lyapunov's direct method refer to Pai (1981), chap. 4; Pai (1989), chap. 1; Fouad and Vittal (1992), chap. 3.

11.1 Review of Literature on Direct Methods

In the review presented here, we will deal only with work relating to the transient stability analysis of multimachine power systems. In this case the simple example presented above becomes quite complex. Several vehicles which correspond to the synchronous machines are now involved. It also becomes necessary to determine (a) which vehicles will be pushed the hardest, (b) what portion of the disturbance energy is distributed to each vehicle, (c) in which directions the vehicles move, and (d) how high a hill must the vehicles climb before they will go over.

Energy criteria for transient stability analysis were the earliest of all direct methods of multimachine power system transient stability assessment. These techniques were extensions of the equal area criterion to power systems with more than two generators represented by the classical model (Anderson and Fouad, 1994, chap. 2). Researchers from the Soviet Union conducted early work in this area (1930s and 1940s). There were very few results on this topic in Western literature during the same period. In the 1960s the application of Lyapunov's direct method to power systems generated a great deal of activity in the academic community. In most of these investigations, the classical power system model was used. The early work on energy criteria dealt with two main issues: (a) characterization of the system energy, and (b) the critical value of the energy.

Several excellent references that provide a detailed review of the development of the direct methods for transient stability exist. Ribbens-Pavella (1971) and Fouad (1975) are early review papers and

provide a comprehensive review of the work done in the period 1960–1975. Detailed reviews of more recent work are conducted in Bose (1984), Ribbens-Pavella and Evans (1985), Fouad and Vittal (1988), and Chiang et al. (1995). The following textbooks provide a comprehensive review and also present detailed descriptions of the various approaches related to direct stability methods: Pai (1981), Pai (1989), Fouad and Vittal (1992), Ribbens-Pavella (1971), and Pavella and Murthy (1994). These references provide a thorough and detailed review of the evolution of the direct methods. In what follows, a brief review of the field and the evolutionary steps in the development of the approaches are presented.

Gorev (1971) first proposed an energy criteria based on the lowest saddle point or unstable equilibrium point (UEP). This work influenced the thinking of power system direct stability researchers for a long time. Magnusson (1947) presented an approach very similar to that of Gorev's and derived a potential energy function with respect to the (posttransient) equilibrium point of the system. Aylett (1958) studied the phase-plane trajectories of multimachine systems using the classical model. An important aspect of this work is the formulation of the system equations based on the intermachine movements. In the period that followed, several important publications dealing with the application of Lyapunov's method to power systems appeared. These works largely dealt with the aspects of obtaining better Lyapunov function, and determining the least conservative estimate of the domain of attraction. Gless (1966) applied Lyapunov's method to the one machine classical model system. El-Abiad and Nagappan (1966) developed a Lyapunov function for multimachine system and demonstrated the approach on a four machine system. The stability results obtained were conservative, and the work that followed this largely dealt with improving the Lyapunov function. A sampling of the work following this line of thought is presented in Willems (1968), Pai et al. (1970), and Ribbens-Pavella (1971). These efforts were followed by the work of Tavora and Smith (1972) dealing with the transient energy of a multimachine system represented by the classical model. They formulated the system equations in the Center of Inertia (COI) reference frame and also in the internode coordinates which is similar to the formulation used by Aylett (1958). Tavora and Smith obtained expressions for the total kinetic energy of the system and the transient kinetic energy, which the authors say determines stability. This was followed by work of Gupta and El-Abiad (1976), which recognized that the UEP of interest is not the one with the lowest energy, but rather the UEP closest to the system trajectory. Uyemura et al. (1996) made an important contribution by developing a technique to approximate the path-dependent terms in the Lyapunov functions by path-independent terms using approximations for the system trajectory.

The work by Athay, Podmore, and colleagues (Athay et al., 1979) is the basis for the transient energy function (TEF) method used today. This work investigated many issues dealing with the application of the TEF method to large power systems. These included:

1. COI formulation and approximation of path-dependent terms.
2. Search for the UEP in the direction of the faulted trajectory.
3. Investigation of the Potential Energy Boundary Surface (PEBS).
4. Application of the technique to power systems of practical sizes.
5. Preliminary investigation of higher-order models for synchronous generators.

This work was followed by the work at Iowa State University by Fouad and colleagues (1981), which dealt with the determination of the correct UEP for stability assessment. This work also identified the appropriate energy for system separation and developed the concept of corrected kinetic energy. Details regarding this work are presented in Fouad and Vittal (1992).

The work that followed largely dealt with developing the TEF method into a more practical tool, and with improving its accuracy, modeling features, and speed. An important development in this area was the work of Bergen and Hill (1981). In this work the network structure was preserved for the classical model. As a result, fast techniques that incorporated network sparsity could be used to solve the problem. A concerted effort was also carried out to extend the applicability of the TEF method to realistic systems. This included improvements in modeling features, algorithms, and computational

efficiency. Work related to the large-scale demonstration of the TEF method is found in Carvalho et al. (1986). The work dealing with extending the applicability of the TEF method is presented in Fouad et al. (1986). Significant contributions to this aspect of the TEF method can also be found in Padiyar and Sastry (1987), Padiyar and Ghosh (1989), and Abu-Elnaga et al. (1988).

In Chiang (1985), Chiang et al. (1987), and Chiang et al. (1988), a significant contribution was made to provide an analytical justification for the stability region for multimachine power systems, and a systematic procedure to obtain the controlling UEP was also developed. Zaborsky et al. (1988) also provide a comprehensive analytical foundation for characterizing the region of stability for multimachine power systems.

With the development of a systematic procedure to determine and characterize the region of stability, a significant effort was directed toward the application of direct methods for online transient stability assessment. This work, reported in Waight et al. (1994) and Chadalavada et al. (1997), has resulted in an online tool which has been implemented and used to rank contingencies based on their severity. Another online approach implemented and being used at B.C. Hydro is presented in Mansour et al. (1995). A recent effort with regard to classifying and ranking contingencies quite similar to the one presented in Chadalavada et al. (1997) is described in Chiang et al. (1998).

Some recent efforts (Ni and Fouad, 1987; Hiskens et al., 1992; Jiang et al., 1995) also deal with the inclusion of FACTS devices in the TEF analysis.

11.2 The Power System Model

The classical power system model will now be presented. It is the “simplest” power system model used in stability studies and is limited to the analysis of first swing transients. For more details regarding the model, the reader is referred to Anderson and Fouad (1994), Fouad and Vittal (1992), Kundur (1994), and Sauer and Pai (1998). The assumptions commonly made in deriving this model are:

For the synchronous generators

1. Mechanical power input is constant.
2. Damping or asynchronous power is negligible.
3. The generator is represented by a constant EMF behind the direct axis transient (unsaturated) reactance.
4. The mechanical rotor angle of a synchronous generator can be represented by the angle of the voltage behind the transient reactance.

The load is usually represented by passive impedances (or admittances), determined from the predisturbance conditions. These impedances are held constant throughout the stability study. This assumption can be improved using nonlinear models. See Fouad and Vittal (1992), Kundur (1994), and Sauer and Pai (1998) for more details. With the loads represented as constant impedances, all the nodes except the internal generator nodes can be eliminated. The generator reactances and the constant impedance loads are included in the network bus admittance matrix. The generators' equations of motion are then given by

$$\begin{aligned} M_i \frac{d\omega_i}{dt} &= P_i - P_{ei} \\ \frac{d\delta_i}{dt} &= \omega_i \quad i = 1, 2, \dots, n \end{aligned} \quad (11.1)$$

where

$$P_{ei} = \sum_{\substack{j=1 \\ j \neq i}}^n [C_{ij} \sin(\delta_i - \delta_j) + D_{ij} \cos(\delta_i - \delta_j)] \quad (11.2)$$

$$\begin{aligned}
P_i &= P_{mi} - E_i^2 G_{ii} \\
C_{ij} &= E_i E_j B_{ij}, \quad D_{ij} = E_i E_j G_{ij} \\
P_{mi} &= \text{Mechanical power input} \\
G_{ii} &= \text{Driving point conductance} \\
E_i &= \text{Constant voltage behind the direct axis transient reactance} \\
\omega_p, \delta_i &= \text{Generator rotor speed and angle deviations, respectively, with respect to a synchronously rotating reference frame} \\
M_i &= \text{Inertia constant of generator} \\
B_{ij} \text{ (} G_{ij} \text{)} &= \text{Transfer susceptance (conductance) in the reduced bus admittance matrix}
\end{aligned}$$

Equation (11.1) is written with respect to an arbitrary synchronous reference frame. Transformation of this equation to the inertial center coordinates not only offers physical insight into the transient stability problem formulation in general, but also removes the energy associated with the motion of the inertial center which does not contribute to the stability determination. Referring to Eq. (11.1), define

$$\begin{aligned}
M_T &= \sum_{i=1}^n M_i \\
\delta_0 &= \frac{1}{M_T} \sum_{i=1}^n M_i
\end{aligned}$$

then,

$$\begin{aligned}
M_T \dot{\omega}_0 &= \sum_{i=1}^n P_i - P_{ei} = \sum_{i=1}^n P_i - 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n D_{ij} \cos \delta_{ij} \\
\dot{\delta}_0 &= \omega_0
\end{aligned} \tag{11.3}$$

The generators' angles and speeds with respect to the inertial center are given by

$$\begin{aligned}
\theta_i &= \delta_i - \delta_0 \\
\tilde{\omega}_i &= \omega_i - \omega_0 \quad i = 1, 2, \dots, n
\end{aligned} \tag{11.4}$$

and in this coordinate system the equations of motion are given by

$$\begin{aligned}
M_i \dot{\tilde{\omega}}_i &= P_i - P_{mi} - \frac{M_i}{M_T} P_{COI} \\
\dot{\theta}_i &= \tilde{\omega}_i \quad i = 1, 2, \dots, n
\end{aligned} \tag{11.5}$$

11.2.1 Review of Stability Theory

A brief review of the stability theory applied to the TEF method will now be presented. This will include a few definitions, some important results, and an analytical outline of the stability assessment formulation.

The definitions and results that are presented are for differential equations of the type shown in Eqs. (11.1) and (11.5). These equations have the general structure given by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t)) \tag{11.6}$$

The system described by Eq. (11.6) is said to be *autonomous* if $\mathbf{f}(t, \mathbf{x}(t)) \equiv \mathbf{f}(\mathbf{x})$, i.e., independent of t and is said to be nonautonomous otherwise.

A point $\mathbf{x}_0 \in R^n$ is called an *equilibrium point* for the system [Eq. (11.6)] at time t_0 if $\mathbf{f}(t, \mathbf{x}_0) \equiv \mathbf{0}$ for all $t \geq t_0$.

An equilibrium point $\mathbf{x}_e$ of Eq. (11.6) is said to be an isolated equilibrium point if there exists some neighborhood S of $\mathbf{x}_e$ which does not contain any other equilibrium point of Eq. (11.6).

Some precise definitions of stability in the sense of Lyapunov will now be presented. In presenting these definitions, we consider systems of equations described by Eq. (11.6), and also assume that Eq. (11.6) possesses an isolated equilibrium point at the origin. Thus, $\mathbf{f}(t, \mathbf{0}) = \mathbf{0}$ for all $t \geq 0$.

The equilibrium $\mathbf{x} = \mathbf{0}$ of Eq. (11.6) is said to be *stable* in the sense of Lyapunov, or simply stable if for every real number $\varepsilon > 0$ and initial time $t_0 > 0$ there exists a real number $\delta(\varepsilon, t_0) > 0$ such that for all initial conditions satisfying the inequality $\|\mathbf{x}(t_0)\| = \|\mathbf{x}_0\| < \delta$, the motion satisfies $\|\mathbf{x}(t)\| < \varepsilon$ for all $t \geq t_0$.

The symbol $\|\cdot\|$ stands for a norm. Several norms can be defined on an n -dimensional vector space. Refer to Miller and Michel (1983) and Vidyasagar (1978) for more details. The definition of stability given above is unsatisfactory from an engineering viewpoint, where one is more interested in a stricter requirement of the system trajectory to eventually return to some equilibrium point. Keeping this requirement in mind, the following definition of asymptotic stability is presented.

The equilibrium $\mathbf{x} = \mathbf{0}$ of Eq. (11.6) is *asymptotically stable* at time t_0 if

1. $\mathbf{x} = \mathbf{0}$ is stable at $t = t_0$
2. For every $t_0 \geq 0$, there exists an $\eta(t_0) > 0$ such that $\lim_{t \rightarrow \infty} \|\mathbf{x}(t)\| \rightarrow 0$ whenever $\|\mathbf{x}(t)\| < \eta$
(ATTRACTIVITY)

This definition combines the aspect of stability as well as attractivity of the equilibrium. The concept is local, because the region containing all the initial conditions that converge to the equilibrium is some portion of the state space. Having provided the definitions pertaining to stability, the formulation of the stability assessment procedure for power systems is now presented. The system is initially assumed to be at a predisturbance steady-state condition governed by the equations

$$\dot{\mathbf{x}}(t) = \mathbf{f}^p(\mathbf{x}(t)) \quad -\infty < t \leq 0 \quad (11.7)$$

The superscript p indicates predisturbance. The system is at equilibrium, and the initial conditions are obtained from the power flow solution. At $t = 0$, the disturbance or the fault is initiated. This changes the structure of the right-hand sides of the differential equations, and the dynamics of the system are governed by

$$\dot{\mathbf{x}}(t) = \mathbf{f}^f(\mathbf{x}(t)) \quad 0 < t \leq t_{cl} \quad (11.8)$$

where the superscript f indicates faulted conditions. The disturbance or the fault is removed or cleared by the protective equipment at time t_{cl} . As a result, the network undergoes a topology change and the right-hand sides of the differential equations are again altered. The dynamics in the postdisturbance or postfault period are governed by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) \quad t_{cl} < t \leq \infty \quad (11.9)$$

The stability analysis is done for the system in the postdisturbance period. The objective is to ascertain asymptotic stability of the postdisturbance equilibrium point of the system governed by Eq. (11.9). This is done by obtaining the domain of attraction of the postdisturbance equilibrium and determining if the initial conditions of the postdisturbance period lie within this domain of attraction or outside it. The domain of attraction is characterized by the appropriately determined value of the transient energy function. In the literature survey presented previously, several approaches to characterize the domain of attraction were mentioned. In earlier approaches (El-Abiad and Nagappan, 1966; Tavora and Smith, 1972), this was done by obtaining the unstable equilibrium points (UEP) of the postdisturbance system

and determining the one with the lowest level of potential energy with respect to the postdisturbance equilibrium. This value of potential energy then characterized the domain of attraction. In the work that followed, it was found that this approach provided very conservative results for power systems. In Gupta and El-Abiad (1976), it was recognized that the appropriate UEP was dependent on the fault location, and the concept of closest UEP was developed. An approach to determine the domain of attraction was also presented by Kakimoto and colleagues (1978; 1981) based on the concept of the potential energy boundary surface (PEBS). For a given disturbance trajectory, the PEBS describes a “local” approximation of the stability boundary. The process of finding this local approximation is associated with the determination of the stability boundary of a lower dimensional system (see Fouad and Vittal [1992], chap. 4 for details). It is formed by joining points of maximum potential energy along any direction originating from the postdisturbance stable equilibrium point. The PEBS constructed in this manner is orthogonal to the equipotential curves. In addition, along the direction orthogonal to the PEBS, the potential energy achieves a local maximum at the PEBS. In Athay et al. (1979), several simulations on realistic systems were conducted. These simulations, together with the synthesis of previous results in the area led to the development of a procedure to determine the correct UEP to characterize the domain of attraction. The results obtained were much improved, but in terms of practical applicability there was room for improvement. The work presented in Fouad et al. (1981) and Carvalho et al. (1986) made several important contributions to determining the correct UEP. The term *controlling UEP* was established, and a systematic procedure to determine the controlling UEP was developed. This will be described later. In Chiang et al. (1985; 1987; 1988), a thorough analytical justification for the concept of the controlling UEP and the characterization of the domain of attraction was developed. This provides the analytical basis for the application of the TEF method to power systems. These analytical results in essence show that the stability boundary of the postdisturbance equilibrium point is made up of the union of the stable manifolds of those unstable equilibrium points contained on the stability boundary. The boundary is then approximated locally using the energy function evaluated at the controlling UEP. The conceptual framework of the TEF approach is illustrated in Fig. 11.2.

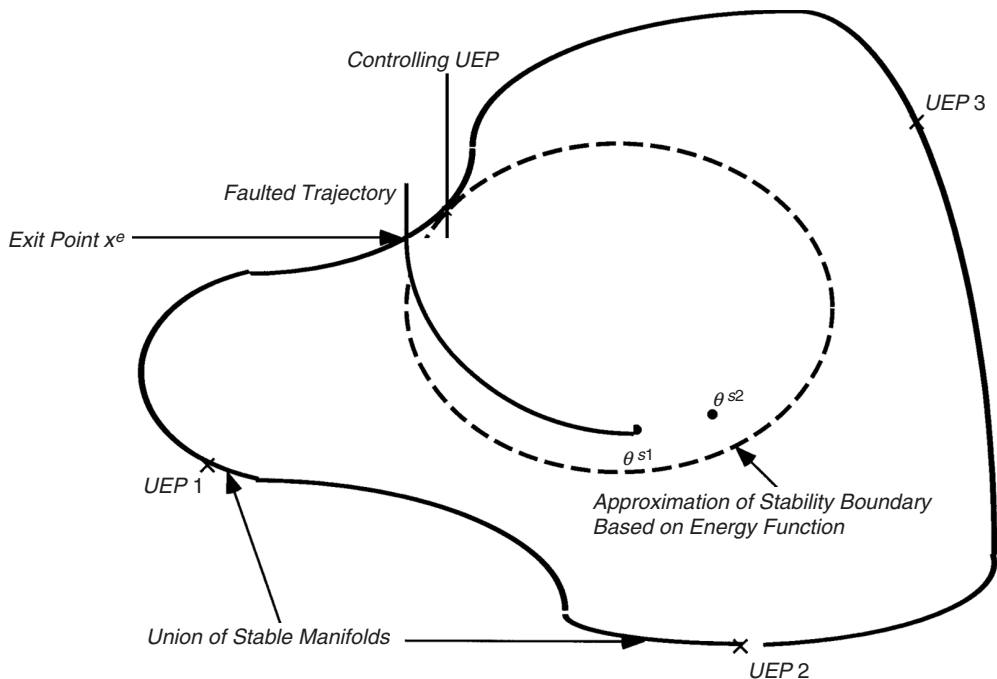


FIGURE 11.2 Conceptual framework of TEF approach.

11.3 The Transient Energy Function

The TEF can be derived from Eq. (11.5) using first principles. Details of the derivation can be found in Pai (1981), Pai (1989), Fouad and Vittal (1992), Athay et al. (1979). For the power system model considered in Eq. (11.5), the TEF is given by

$$V = \frac{1}{2} \sum_{i=1}^n M_i \tilde{\omega}_i^2 - \sum_{i=1}^n P_i (\theta_i - \theta_i^{s2}) - \sum_{i=1}^{n-1} \sum_{j=i+1}^n \left[C_{ij} \cos(\theta_{ij} - \theta_{ij}^{s2}) - \int_{\theta_i^{s2} + \theta_j^{s2}}^{\theta_i + \theta_j} D_{ij} \cos \theta_{ij} d(\theta_i + \theta_j) \right] \quad (11.10)$$

where $\theta_{ij} = \theta_i - \theta_j$.

The first term on the right-hand side of Eq. (11.10) is the kinetic energy. The next three terms represent the potential energy. The last term is path dependent. It is usually approximated (Uyemura et al., 1996; Athay et al., 1979) using a straight line approximation for the system trajectory. The integral between two points θ^a and θ^b is then given by

$$I_{ij} = D_{ij} \frac{\theta_i^b - \theta_i^a + \theta_j^b - \theta_j^a}{\theta_{ij}^b - \theta_{ij}^a} (\sin \theta_{ij}^b - \sin \theta_{ij}^a). \quad (11.11)$$

In Fouad et al. (1981), a detailed analysis of the energy behavior along the time domain trajectory was conducted. It was observed that in all cases where the system was stable following the removal of a disturbance, a certain amount of the total kinetic energy in the system was not absorbed. This indicates that not all the kinetic energy created by the disturbance, contributes to the instability of the system. Some of the kinetic energy is responsible for the intermachine motion between the generators and does not contribute to the separation of the severely disturbed generators from the rest of the system. The kinetic energy associated with the gross motion of k machines having angular speeds $\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_k$ is the same as the kinetic energy of their inertial center. The speed of the inertial center of that group and its kinetic energy are given by

$$\tilde{\omega}_{cr} = \sum_{i=1}^k M_i \tilde{\omega}_i / \sum_{i=1}^k M_i \quad (11.12)$$

$$V_{KE_{cr}} = \frac{1}{2} \left[\sum_{i=1}^k M_i \right] (\tilde{\omega}_{cr})^2 \quad (11.13)$$

The disturbance splits the generators of the system into two groups: the critical machines and the rest of the generators. Their inertial centers have inertia constants and angular speeds $M_{cr}, \tilde{\omega}_{cr}$ and $M_{sys}, \tilde{\omega}_{sys}$, respectively. The kinetic energy causing the separation of the two groups is the same as that of an equivalent one-machine-infinite-bus system having inertia constant M_{eq} and angular speed $\tilde{\omega}_{eq}$ given by

$$\begin{aligned} M_{eq} &= \frac{M_{cr} \times M_{sys}}{M_{eq} + M_{sys}} \\ \tilde{\omega}_{eq} &= (\tilde{\omega}_{cr} - \tilde{\omega}_{sys}) \end{aligned} \quad (11.14)$$

and the corresponding kinetic energy is given by

$$V_{KE_{corr}} = \frac{1}{2} M_{eq} (\tilde{\omega}_{eq})^2 \quad (11.15)$$

The kinetic energy term in Eq. (11.10) is replaced by Eq. (11.15).

11.4 Transient Stability Assessment

As described previously, the transient stability assessment using the TEF method is done for the final postdisturbance configuration. The stability assessment is done by comparing two values of the transient energy V . The value of V is computed at the end of the disturbance. If the disturbance is a simple fault, the value of V at fault clearing V_{cl} is evaluated.

The other value of V that largely determines the accuracy of the stability assessment is the critical value of V , V_{cr} which is the potential energy at the controlling UEP for the particular disturbance being investigated.

If $V_{cl} < V_{cr}$ the system is stable, and if $V_{cl} > V_{cr}$ the system is unstable. The assessment is made by computing the energy margin ΔV given by

$$\Delta V = V_{cr} - V_{cl} \quad (11.16)$$

Substituting for V_{cr} and V_{cl} from Eq. (11.10) and invoking the linear path assumption for the path dependent integral between the conditions at the end of the disturbance and the controlling UEP, we have

$$\begin{aligned} \Delta V = & -\frac{1}{2} M_{eq} \tilde{\omega}_{eq}^{cl^2} - \sum_{i=1}^n P_i (\theta_i^u - \theta_i^{cl}) \\ & - \sum_{i=1}^{n-1} \sum_{j=i+1}^n \left[C_{ij} (\cos \theta_{ij}^u - \cos \theta_{ij}^{cl}) \right] - D_{ij} \frac{\theta_i^u - \theta_i^{cl} + \theta_j^u - \theta_j^{cl}}{(\theta_{ij}^u - \theta_{ij}^{cl})} (\sin \theta_{ij}^u - \sin \theta_{ij}^{cl}) \end{aligned} \quad (11.17)$$

where $(\theta^{cl}, \tilde{\omega}^{cl})$ are the conditions at the end of the disturbance and $(\theta^u, \mathbf{0})$ represents the controlling UEP. If ΔV is greater than zero the system is stable, and if ΔV is less than zero, the system is unstable. A qualitative measure of the degree of stability (or instability) can be obtained if ΔV is normalized with respect to the corrected kinetic energy at the end of the disturbance (Fouad et al., 1981).

$$\Delta V_n = \Delta V / V_{KE_{corr}} \quad (11.18)$$

For a detailed description of the computational steps involved in the TEF analysis, refer to Fouad and Vittal (1992), chap. 6.

11.5 Determination of the Controlling UEP

A detailed description of the rationale in developing the concept of the controlling UEP is provided in Fouad and Vittal (1992), section 5.4. A criterion to determine the controlling UEP based on the normalized energy margin is also presented. The criterion is stated as follows. The postdisturbance trajectory approaches (if the disturbance is large enough) the controlling UEP. This is the UEP with the lowest normalized potential energy margin. The determination of the controlling UEP involves the following key steps:

1. Identifying the correct UEP.
2. Obtaining a starting point for the UEP solution close to the exact UEP.
3. Calculation of the exact UEP.

Identifying the correct UEP involves determining the advanced generators for the controlling UEP. This is referred to as the mode of disturbance (MOD). These generators generally are the most severely disturbed generators due to the disturbance. The generators in the MOD are not necessarily those that lose synchronism. The computational details of the procedure to identify the correct UEP and obtain a

starting point for the exact UEP solution are provided in Fouad and Vittal (1992), section 6.6. An outline of the procedure is provided below:

1. Candidate modes to be tested by the MOD test depend on how the disturbance affects the system. The selection of the candidate modes is based on several disturbance severity measures obtained at the end of the disturbance. These severity measures include kinetic energy and acceleration. A ranked list of machines is obtained using the severity measures. From this ranked list, the machines or group of machines at the bottom of the list are included in the group forming the rest of the system and V_{KEcorr} is calculated. In a sequential manner, machines are successively added to the group forming the rest of the system and V_{KEcorr} is calculated and stored.
2. The list of V_{KEcorr} calculated above is sorted in descending order and only those groups within 10% of the maximum V_{KEcorr} in the list are retained.
3. Corresponding to the MOD for each of the retained groups of machines in step 2, an approximation to the UEP corresponding to that mode is constructed using the postdisturbance stable equilibrium point. For a given candidate mode, where machines i and j are contained in the critical group, an estimate of the approximation to the UEP for an n -machine system is given by $\left[\hat{\theta}_{ij}^u\right]^T = \left[\theta_1^{s2}, \theta_2^{s2}, \dots, [\pi - \theta_i^{s2}], \dots, [\pi - \theta_j^{s2}], \dots, \theta_n^{s2}\right]$. This estimate can be further improved by accounting for the motion of the COI, and using the concept of the PEBS to maximize the potential energy along the ray drawn from the estimate and the postdisturbance stable equilibrium point θ^{s2} .
4. The normalized potential energy margin for each of the candidate modes is evaluated at the approximation to the exact UEP, and the mode corresponding to the lowest normalized potential energy margin is then selected as the mode of the controlling UEP.
5. Using the approximation to the controlling UEP as a starting point, the exact UEP is obtained by solving the nonlinear algebraic equation given by

$$f_i = P_i - P_{mi} - \frac{M_i}{M_T} P_{COI} = 0 \quad i = 1, 2, \dots, n \quad (11.19)$$

The solution of these equations is a computationally intensive task for realistic power systems. Several investigators have made significant contributions to determining an effective solution. A detailed description of the numerical issues and algorithms to determine the exact UEP solution are beyond the scope of this handbook. Several excellent references that detail these approaches are available. These efforts are described in Fouad and Vittal (1992), section 6.8.

11.6 The BCU (Boundary Controlling UEP) Method

The BCU method (Chiang et al., 1985, 1987, 1988) provides a systematic procedure to determine a suitable starting point for the controlling UEP solution. The main steps in the procedure are as follows:

1. Obtain the faulted trajectory by integrating the equations

$$\begin{aligned} M_i \dot{\tilde{\omega}}_i &= P_i^f - P_{ei}^f - \frac{M_i}{M_T} P_{COI}^f \\ \dot{\tilde{\theta}}_i &= \tilde{\omega}_i, \quad i = 1, 2, \dots, n \end{aligned} \quad (11.20)$$

Values of θ obtained from Eq. (11.20) are substituted in the postfault mismatch equation given by Eq. (11.19). The exit point $\mathbf{x}^e$ is then obtained by satisfying the condition $\sum_{i=1}^n -f_i \tilde{\omega}_i = 0$.

- Using θ^e as the starting point, integrate the associated gradient system equations given by

$$\begin{aligned}\dot{\theta}_i &= P_i - P_{ei} - \frac{M_i}{M_T} P_{COI}, \quad i = 1, 2, \dots, n-1 \\ \theta_n &= - \sum_{i=1}^{n-1} M_i \theta_i / M_n\end{aligned}\tag{11.21}$$

At each step of the integration, evaluate $\sum_{i=1}^n |f_i| = F$ and determine the first minimum of F along the gradient surface. Let θ^* be the vector of rotor angles at this point.

- Using θ^* as a starting point in [Eq. \(11.19\)](#), obtain the exact solution for the controlling UEP.

11.7 Applications of the TEF Method and Modeling Enhancements

The preceding subsections have provided the important steps in the application of the TEF method to analyze the transient stability of multimachine power systems. In this subsection, a brief mention of the applications of the technique and enhancements in terms of modeling detail and application to realistic power systems is provided. Inclusion of detailed generator models and excitation systems in the TEF method are presented in Athay et al. (1979), Fouad et al. (1986), and Waight et al. (1994). The sparse formulation of the system to obtain more efficient solution techniques is developed in Bergen and Hill (1981), Abu-Elnaga et al. (1988), and Waight et al. (1994). The application of the TEF method for a wide range of problems including dynamic security assessment are discussed in Fouad and Vittal (1992), chaps. 9–10; Chadalavada et al. (1997); and Mansour et al. (1995). The availability of a qualitative measure of the degree of stability or instability in terms of the energy margin makes the direct methods an attractive tool for a wide range of problems. The modeling enhancements that have taken place and the continued development in terms of computational efficiency and computer hardware, make direct methods a viable candidate for online transient stability assessment (Waight et al., 1994; Chadalavada et al., 1997; Mansour et al., 1995). This feature is particularly effective in the competitive market environment to calculate operating limits with changing conditions. There are several efforts underway dealing with the development of direct methods and a combination of time simulation techniques for online transient stability assessment. These approaches take advantage of the superior modeling capability available in the time simulation engines, and use the qualitative measure provided by the direct methods to derive preventive and corrective control actions and estimate limits. This line of investigation has great potential and could become a vital component of energy control centers in the near future.

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